CHAPTER 14

FLUIDS

OVERVIEW:

The physics of fluids is the basic of hydraulic engineering. That has a vast of applications such as a medical engineering might study the blood flow in arteries of an aging patient, an environment engineer might be concerned about the drainage from waste sites or the efficient irrigation of farmlands, a naval engineer might be concerned with the dangers faced by a deep sea diver, etc.

WHAT IS A FLUID?

There are four primary states of matter: ***liquid, gas, solid, and plasma***

Plasma examples:

- aurorae.
- the excited low-pressure gas inside neon signs and fluorescent lights.
- solar wind.
- welding arcs.
- the Earth's ionosphere.
- stars (including the Sun)
- the tail of a comet.
- lightning

The fifth state (man-made Bose-Einstein condensate) .

A fluid, in contrast to a solid, is a substance that can flow: liquids and gases

A fluid can flow, assume the shape of a container, if placed in a sealed container, will distribute applied pressure evenly surface in the container. A liquid is nearly incompressible unlike gas.

Density and Pressure:

For solids, we are interested more in mass and forces. With fluids, we are more interested in density and pressure.

To find the density ρ of a fluid at nay points, we isolate a small volume element ΔV around that point and measure the mass Δm of the fluid contained within that element. The density is then:

$$\rho = \frac{\Delta m}{\Delta V}$$

In practice, we assume that a fluid sample is large relative to atomic dimensions and thus is smooth, uniform density, rather than lumpy with atoms. This assumption allows us to write:

$$\boldsymbol{\rho = \frac{m}{V}}$$

Density is a scalar property. Its unit is the kilogram per cubic meter. [kg/m^3]

Density of water = 998 kg/m^3 ~ 1000 kg/m^3

Note that the density of a gas varies considerably with pressure, but the density of a liquid does not.

Pressure: We define the pressure on the piston from the fluid as

$$p = \frac{\Delta F}{\Delta A}$$

Pressure of uniform force on flat area is

$$\boldsymbol{p = \frac{F}{A}} \quad \textbf{[Pa = N/m}^2\textbf{]}$$

Pressure is a scalar, having no directional properties (F is only the magnitude of the force)

Unit of pressure is Pascal [Pa] in SI: **1 atm ~ 1.01*10^5 Pa**

The atmosphere (atm) is the approximate average pressure of the atmosphere at sea level. Other units of pressure:

(1 atm = 760 **mmHg** (**Torr**); 1 atm ~ 1.01 **bar**; 1atm ~ 15 **psi**)

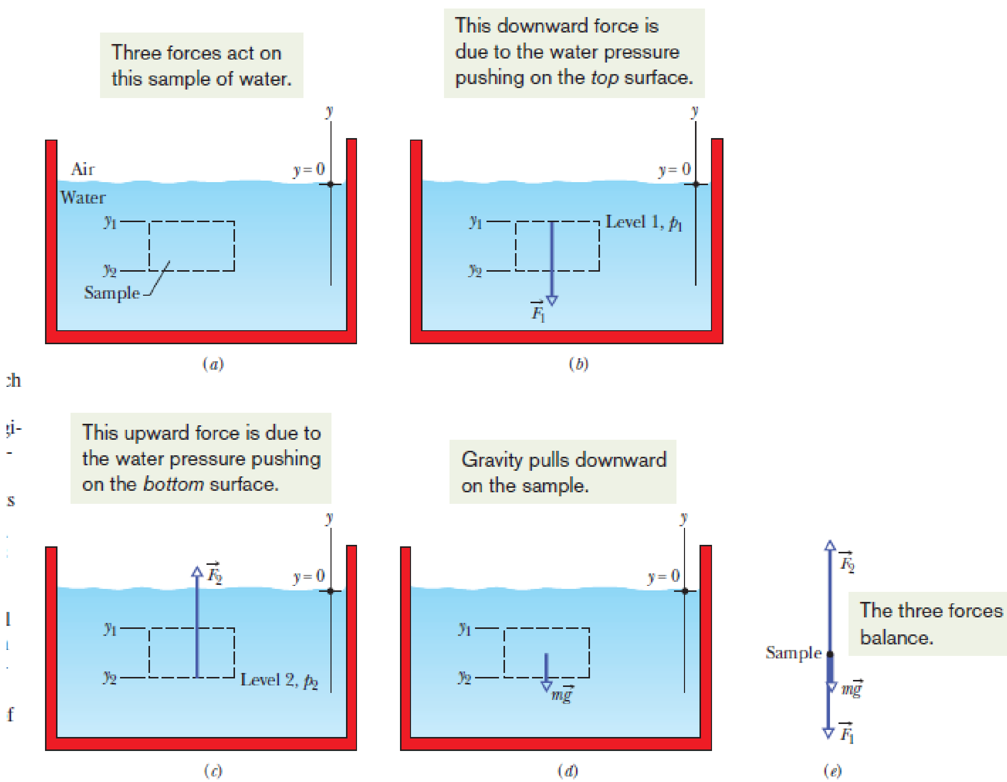
Pressure on mercury to make it raise 760 mm = 0.76 m is:

13,560*9.81*0.76 = 101098 Pa ~ 1 atm.

On the top of Mount Everest 1/3 atm; at the bottom of ocean 1000 atm.

Part 1: FLUID AT REST (fluid statics)

A tank of water opens to the atmosphere. Divers know that the pressure increases with depth below the air. Mountaineers know that the pressure decreases with attitude as one ascends into the atmosphere. The pressures encountered by these divers or mountaineers are usually called hydrostatic pressures due to fluids that are static (at rest).



The balance of these three forces is written as

$$F_2 = F_1 + mg$$

But $\quad F_1 = p_1.A$ and $F_2 = p_2.A \quad$ so $\quad p_2.A = p_1.A + \rho Vg$

Where the volume V is the product of its face area A and its height $y_1 - y_2$

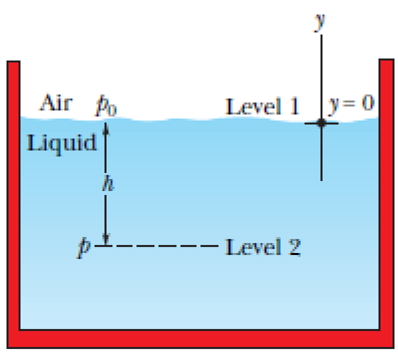
Thus $\quad p_2 = p_1 + \rho g(y_1 - y_2)$

This equation can be used to find pressure both in a liquid (as a function of depth) and in the atmosphere (as a function of height). If we seek the pressure p at a depth h below the liquid surface, we can choose level 1 to be the surface, level 2 to be a distance h below it, and p_o to represent the atmospheric pressure on the surface, then we can write: $\mathbf{p = p_o + \rho_{fluid}\, gh}$

For the atmospheric (uniform over the distance): $\mathbf{p = p_o - \rho_{air} gd}$

if not uniform $p \sim p_o exp(-0.00012h)$ **[Pa]**

Example: Estimate air pressure on the top of Mount Everest (h =8848 m)



Note that the pressure at a given depth in the liquid depends on that depth but not on any horizontal dimension of the fluid or its container.

That equation holds no matter what the shape of the container.

"p" is said to be the ***total pressure, or absolute pressure*** at level 2 because the pressure p at level 2 consists of two contributions p_o due to the atmosphere, which bears down on the liquid, and ρgh - the pressure due to the liquid above level 2, which bears down on level 2. In general, the difference between an absolute pressure and an atmospheric pressure is called the ***gauge pressure***.

The Gauge pressure is $\mathbf{p_g = p - p_o = \rho_{fluid}\, gh}$

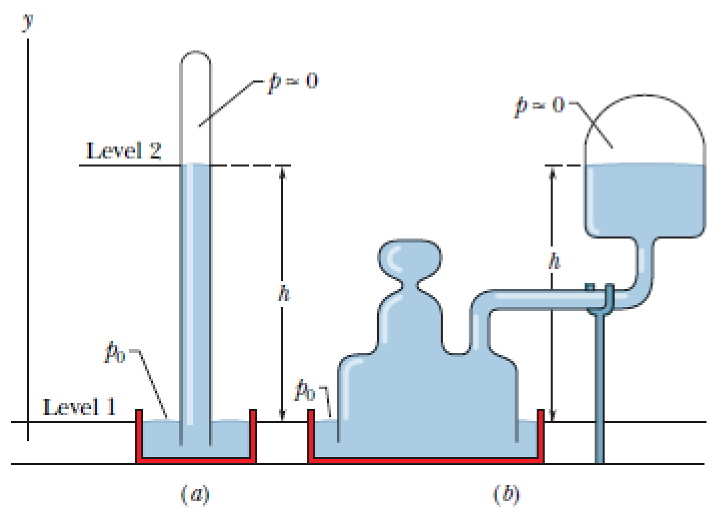
Measuring Pressure:

A very basic mercury barometer is a device to measure the pressure of the atmosphere. If we choose level 1 to be that of the air-mercury interface and level 2 to be that of the top of the mercury column, the we have:

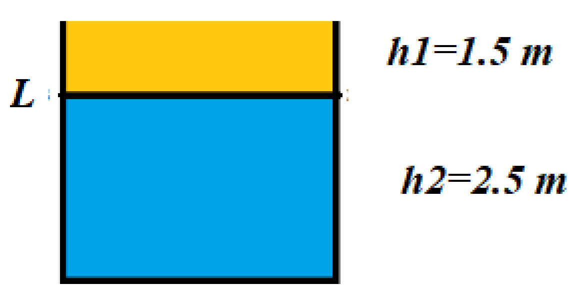
$$p_o = \rho_{Hg}\, gh$$

Where ρ_{Hg} is the density of the mercury. For a given pressure, the height h of the mercury column does not depend on the cross-sectional area of the vertical tube. All that counts is the vertical distance h between the mercury levels.

(*a*) A mercury barometer. (*b*) Another mercury barometer. The distance *h* is the same in both cases.



Example: A tank contains oil and water as below figure. Calculate pressure at level L and at the bottom of the tank if density of water is 998 kg/m^3 and density of oil is 798 kg/m^3; h_1=1.50 m and h_2=2.50 m.



(p_L=113 kPa and p_b=137 kPa)

Example: A scuba diver practicing in a swimming pool takes enough air from his tank to fully expand his lungs before abandoning the tank at depth L and swimming to the surface. He ignores instructions and fails to exhale during his ascent. When he reaches the surface, the difference between the external pressure on him and the air pressure in his lungs is 9.3 kPa. From what depth does he start? What potentially lethal danger does he face?

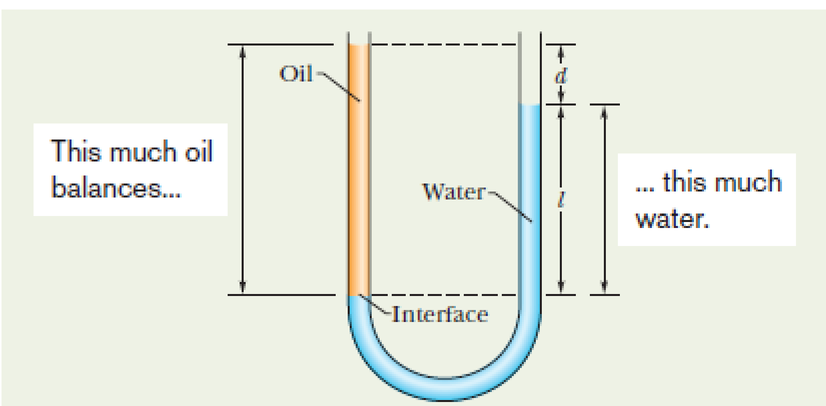
Solution: The pressure at depth h in a liquid of density ρ is given by $p = p_o + \rho gh$

At depth h the external pressure on him and thus the air pressure within his lungs is $p = p_o + \rho gh$ we can find

$$h = \frac{p - p_o}{\rho g} = \frac{9300\ Pa}{\left(998 \frac{kg}{m^3}\right)(9.8\ m/m^2)} = 0.95\ m$$

This is not deep, but the pressure difference is 9.3 kPa. This is sufficient to rupture the diver's lungs and force air from them into the depressurized blood, which then carries the air to the heart.

Example: A U-tube contains two liquids in static equilibrium. Water of density $\rho_w = 998\ kg/m^3$ is in the right arm, and oil of unknown density ρ_{oil} is in the left. Measurement gives l = 135 mm and d=12.3 mm. What is the density of the oil?



Solution:

Because the water is in static equilibrium, pressure at interface position must be the same for right arm and left arm:

$P_{int} = p_o + \rho_w gl = p_o + \rho_{oil} g(l+d)$ therefore

$$\frac{\rho_{oil}}{\rho_w} = \frac{l}{l+d} \quad or \quad \rho_{oil} = (998)\frac{135}{135+12.3} = 915\ kg/m^3$$

Demonstrating Pascal's Principle

Consider the case in which the incompressible fluid is a liquid contained in a tall cylinder, as in Fig. 14-7. The cylinder is fitted with a piston on which a container of lead shot rests. The atmosphere, container, and shot exert pressure p_{ext} on the piston and thus on the liquid. The pressure p at any point P in the liquid is then

$$p = p_{ext} + \rho g h. \quad (14\text{-}11)$$

Let us add a little more lead shot to the container to increase p_{ext} by an amount Δp_{ext}. The quantities ρ, g, and h in Eq. 14-11 are unchanged, so the pressure change at P is

$$\Delta p = \Delta p_{ext}. \quad (14\text{-}12)$$

This pressure change is independent of h, so it must hold for all points within the liquid, as Pascal's principle states.

Pascal's Principle and the Hydraulic Lever

Figure 14-8 shows how Pascal's principle can be made the basis of a hydraulic lever. In operation, let an external force of magnitude F_i be directed downward on the left-hand (or input) piston, whose surface area is A_i. An incompressible liquid in the device then produces an upward force of magnitude F_o on the right-hand (or output) piston, whose surface area is A_o. To keep the system in equilibrium, there must be a downward force of magnitude F_o on the output piston from an external load (not shown). The force $\vec{F}_i$ applied on the left and the downward force $\vec{F}_o$ from the load on the right produce a change Δp in the pressure of the liquid that is given by

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o},$$

so

$$F_o = F_i \frac{A_o}{A_i}. \quad (14\text{-}13)$$

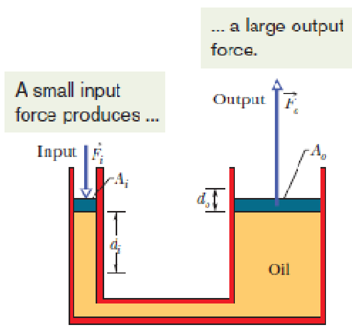


Fig. 14-8 A hydraulic arrangement that can be used to magnify a force $\vec{F}_i$. The work done is, however, not magnified and is the same for both the input and output forces.

The work W done on the input piston by the applied force is equal to the work W done by the output piston in lifting the load placed on it.

$$W = F_o d_o = F_i d_i$$

The advantage of a hydraulic lever is: With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.

Mechanical advantage of a simple machine is $\quad \boldsymbol{M.A.} = \frac{F_o}{F_i}$

Buoyant force:

The buoyancy force is caused by the pressure exerted by the fluid in which an object is immersed.

The buoyancy force always points upwards because the pressure of a fluid increases with depth.

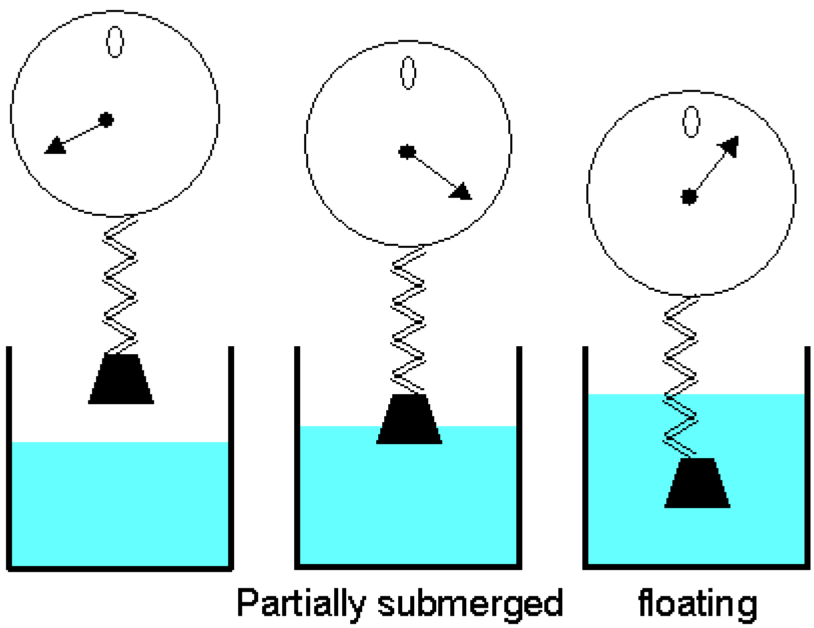
We can calculate the buoyancy force directly by computing the force exerted on each of the object's surfaces, or indirectly by finding the weight of the displaced fluid.

$$F_B = F_{bottom} - F_{top}$$

If the buoyant force is greater than the object's weight, the object rises to the surface and floats. If the buoyant force is less than the object's weight, the object sinks. If the buoyant force equals the object's weight, the object can remain suspended at its present depth. The buoyant force is always present, whether the object floats, sinks, or is suspended in a fluid.

Apparent weight: If we place a stone on a scale that is calibrated to measure weight, the reading on the scale is the stone's weight. However, if we do this underwater, the upward buoyant force on the stone from the water decreases the reading. That reading is then an apparent weight:

Apparent weight = Actual weight – magnitude of buoyant force



ARCHIMEDES' PRINCIPLE:

When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $\boldsymbol{m_f g}$ of the fluid that has been displaced by the body.

$$F_b = m_f g$$

where $\boldsymbol{m_f}$ is the mass of the fluid that is displaced by the body.

$$\textbf{Fraction of submerged} = \frac{\rho_{obj}}{\rho_{fl}} = \frac{h}{H}$$

H is the high of the object and h is the depth of submerged part.

Example: An iceberg (ρ=917 kg/m^3) floats in seawater (ρ=1025 kg/m^3). What fraction of the iceberg is beneath the surface of the water?

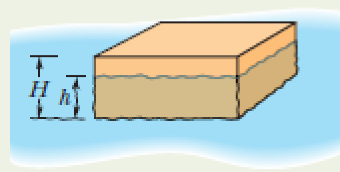
Floating:

When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body. :

$$\mathbf{F_b = F_g}$$

Example: A block of density $\rho = 800$ kg/m^3 floats face down in a fluid of density $\rho_f = 1200$ kg/m^3. The block has height H=6.0 cm.

a. By what depth h is the block submerged?
b. If the block is held fully submerged and then released, what is the magnitude of its acceleration?



Block of height H floats in a fluid, to a depth of h.

Solution:

The block is stationary, thus: **$F_b = F_g$**

where $F_b = m_f g = \rho_f V_f g = \rho_f LWhg$ and $F_g = \rho LWHg$

Solve for h: $\frac{\rho}{\rho_f} = \frac{h}{H}$ so $h = \frac{\rho}{\rho_f} H = \frac{800}{1200} 6.0 = 4.0\ cm$

With the block fully submerged, the volume of the displaced water is V = LWH (full block)

This means that the value of buoyant force is larger, and the block no longer be stationary but will accelerate upward. Newton's second law now:

$$F_b - F_g = ma \quad or \quad \rho_f LWHg - \rho LWHg = \rho LWHa$$

Solve for a: $a = \left(\frac{\rho_f}{\rho} - 1\right) g = \left(\frac{1200}{800} - 1\right)(9.8) = 4.9\ m/s^2$

Part 2: FLOW OF IDEAL FLUIDS: (fluid dynamics)

An ideal fluid is incompressible and lacks viscosity, and its flow is steady and irrotational.

A streamline is the path followed by an individual fluid particle. A tube of flow is a bundle of streamlines. The flow within any tube of flow obeys the

Equation of Continuity: **$R_v = Av$ = a constant**

In which R_v is the volume flow rate, A is the cross-sectional area of the tube of flow at any point, and v is the speed of the fluid at that point.

Mass flow rate is **$R_m = \rho R_v = \rho Av$ = a constant**

Example:

The figure shows how the stream of water emerging from a faucet "neck down" as it falls. This change in the horizontal cross-sectional area is characteristic of any laminar (non turbulent) falling stream because the gravitational force increases the speed of the stream. The indicated cross-sectional areas are $A_1 = 1.2\ cm^2$ and $A = 0.35\ cm^2$.

The two levels are separated by a vertical distance h=45 mm. What is the volume flow rate from the tap?

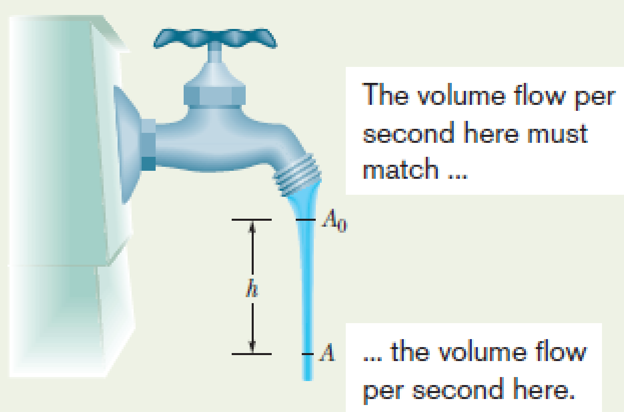


Fig. 14-18 As water falls from a tap, its speed increases. Because the volume flow rate must be the same at all horizontal cross sections of the stream, the stream must "neck down" (narrow).

Solution:

Solve the system of two equations: $A_o v_o = Av \quad and \quad v^2 = v_o^2 + 2gh$

We have $v_o = \sqrt{\frac{2ghA^2}{A_o^2 - A^2}} = \sqrt{\frac{2(9.8)(0.045)(0.35^2)}{(1.2^2 - 0.35^2)}} = 28.6\ cm/s$

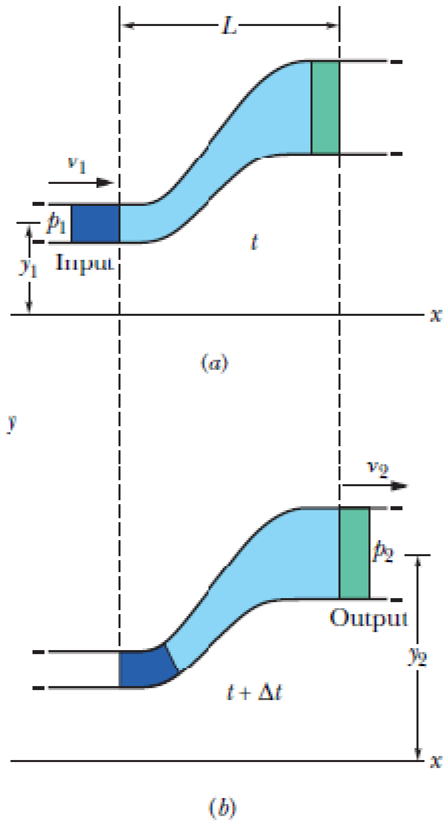
The ***volume flow rate*** is $R_v = A_o v_o = (1.2)(28.6) = 34\ cm^3/s$

BERNOULLI'S EQUATION:

Applying the principle of conservation of mechanical energy to the flow of an ideal fluid leads to Bernoulli's equation

$$p + \frac{1}{2}\rho v^2 + \rho g y = a\ constant \quad (1)$$

Proof:



Fluid flows at a steady rate through a length L of a tube, from the input end at the left to the output end at the right. From time t in (a) to time $t + \Delta t$ in (b), the amount of fluid shown in purple enters the input end and the equal amount shown in green emerges from the output end.

The above figure represents a tube through which an ideal fluid is flowing at a steady rate. In a time interval t, suppose that a volume of fluid V, colored purple enters the tube at the left end and an identical volume colored green emerges at the right end. The emerging volume must be the same because the fluid is incompressible.

Let y_1, v_1 and p_1 be the elevation, speed, and pressure of the fluid entering at the left, and y_2, v_2, and p_2 be the corresponding quantities for the fluid emerging at the right. The fluid lying between the two vertical planes separated by a distance L does not change its properties during this process. We need be concerned only with changes that take place at the input and output ends.

Apply the energy conservation in the form of the work-kinetic energy theorem:

$$W = \Delta K$$

This tells us that the change in the kinetic energy of the system must equal the net work done on the system. The change in kinetic energy results from the change in speed between the ends of the tube and is

$$\Delta K = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$$

Δm is the mass of the fluid that enters at the left end and leaves at the right end during a small time interval Δt.

The work done on the system arises from two sources:

$$\boldsymbol{W = W_g + W_p = \Delta K}$$

a. The work W_g done by the gravitational force ($\Delta m\vec{g}$) on the fluid of mass Δm during the vertical lift of the mass from input level to the output level is:

$$W_g = -\Delta m g(y_2 - y_1) = -\boldsymbol{\rho g}\Delta(\boldsymbol{y_2 - y_1})$$

The work is negative because the upward displacement and the downward gravitational force have opposite directions.

b. The work done on the system at the input end to push the entering fluid into the tube and by the system at the output end to push forward the fluid that is located ahead of the emerging fluid. In general, the work done by a force of magnitude F, acting on a fluid sample contained in a tube of area ΔA to move the fluid through a distance Δx is:

$$F\Delta x = (pA)(\Delta x) = p(A\Delta x) = p\Delta V$$

The work done on the system is then $p_1 \Delta V$ and the work done by the system is $-p_2 \Delta V$. Their sum W_p is $\quad W_p = -p_2\Delta V + p_1\Delta V = -(\boldsymbol{p_2 - p_1})\Delta\boldsymbol{V}$

The work-kinetic energy theorem now becomes:

$$W_g + W_p = \Delta K$$

Or $\quad -\rho g\Delta V(y_2 - y_1) - \Delta V(p_2 - p_1) = \frac{1}{2}\rho\Delta V(v_2^2 - v_1^2)$

This leads to the Bernoulli's equation (1).

Example: In the old West, a desperado fires a bullet into an open water tank, creating a hole a distance h below the water surface. What is the speed v of the water exiting the tank?

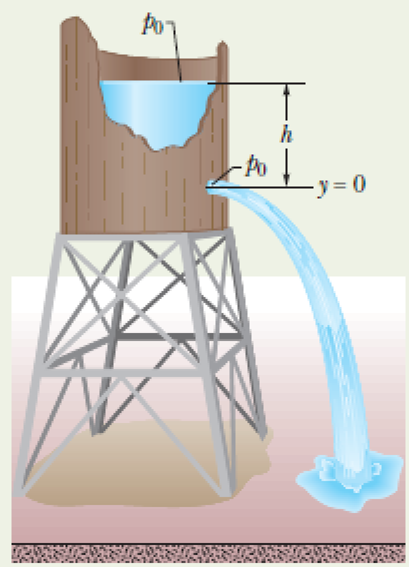


Fig. 14-20 Water pours through a hole in a water tank, at a distance h below the water surface. The pressure at the water surface and at the hole is atmospheric pressure p_0.

Solution:

Solve the system of two equations: $R_v = av = Av_o$

$$and \quad p_o + \frac{1}{2}\rho v_o^2 + \rho gh = p_o + \frac{1}{2}\rho v^2 + \rho g(0)$$

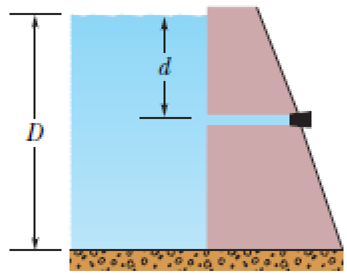
Since a << A, so $\mathbf{v_o} << \mathbf{v}$ so the middle term on the left can be negligible relative to the middle term to the right. Finally, we have: $\boldsymbol{v = \sqrt{2gh}}$

(similar to the free-fall)

Example:

The fresh water behind a reservoir dam has depth D = 15 m. A horizontal pipe 4.0 cm in diameter passes through the dam at depth d=6.0 m. A plug secures the pipe opening. Find the magnitude of the frictional force between plug and pipe wall?

If the plug is removed, what water volume exits the pipe in 3.0 hours?



<u>Solution</u>:

The friction force must at least equal to the force applied on the plus, therefore

$f = A\Delta p = \rho_w g d A = (1000)(9.8)(6)Л(0.02)^2 = 74$ N.

The speed of water flowing out of the hole is $v = \sqrt{2gd}$

The volume of water flowing out of the pipe in t = 3.0 h is

$V = R_v t = (Av)t = 1.5*10^2\ m^3$